



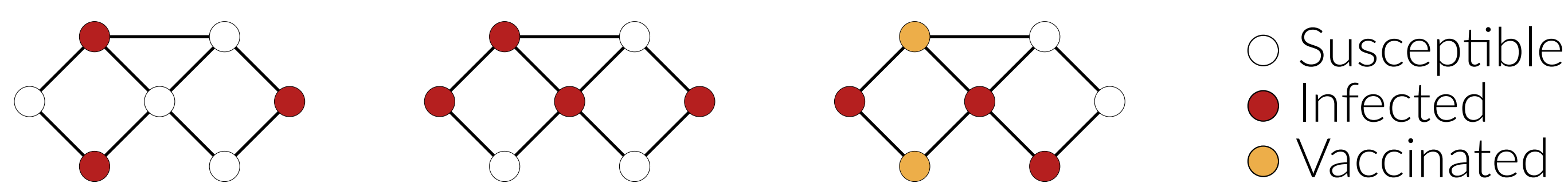
## Motivation

Mathematically rigorous approach to contact network recovery and vaccination in a reinfecting epidemic model, posing two **challenges**:

- **Unknown Network**: The underlying contact graph showing *who can infect whom* is unknown
- **Limited Resources**: We can only vaccinate a small fraction of the population, so vaccinations must be precise and impactful

## The Propagation Model: Graphical SIS

- Individuals represented as vertices in a graph  $\mathcal{G}$  with states Susceptible or Infected
- Edges represent all *potential* infection pathways
- Recovery (**I**  $\rightarrow$  **S**) with probability  $p_{rec}$
- Infection (**S**  $\rightarrow$  **I**) with probability  $\propto p_{inf}$  if neighbors are infected
- Vaccinated vertices: lower probability of infection



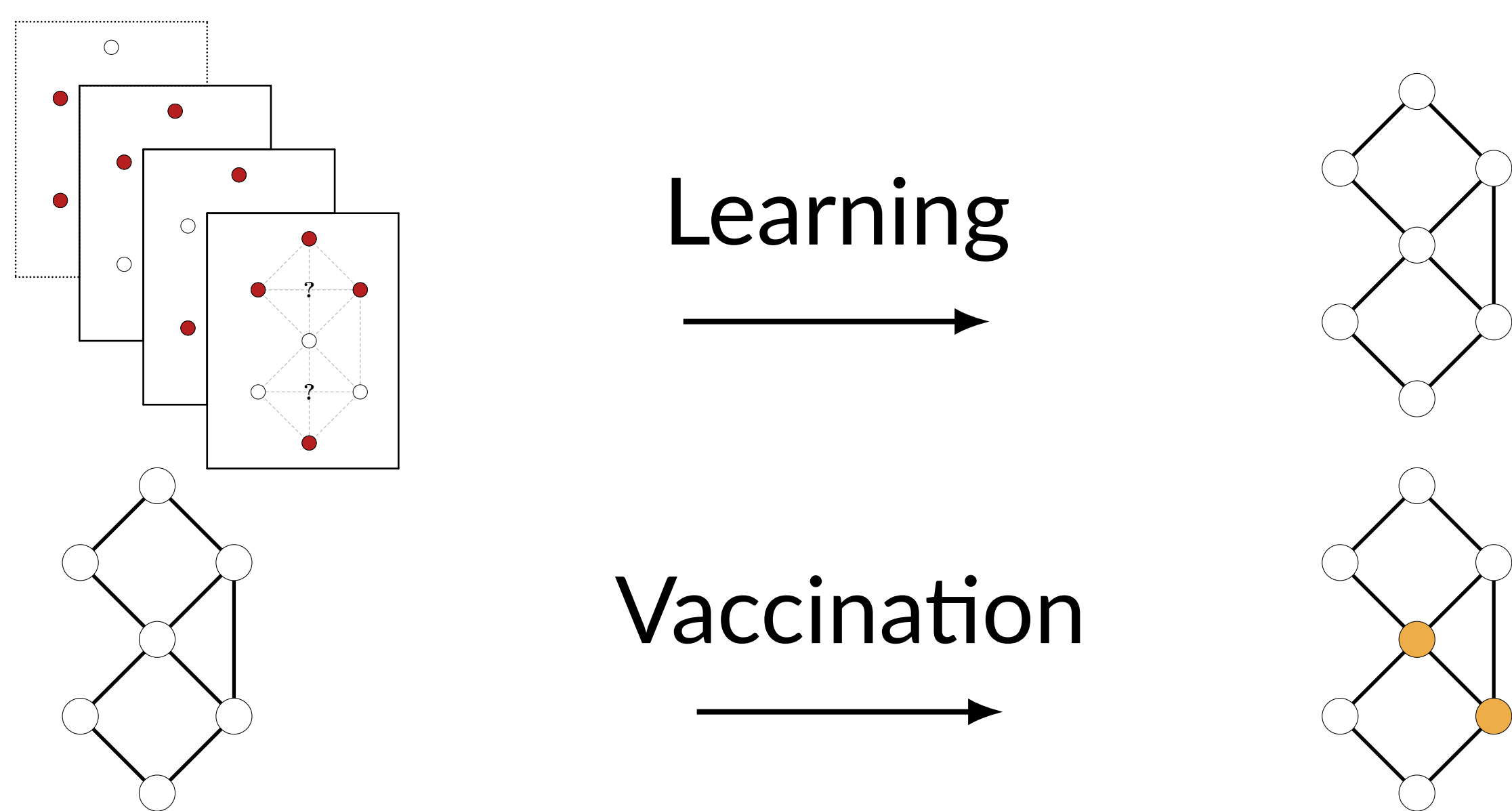
## Vaccinating an Unknown Graph (VUG) Problem

**Goal**: Vaccinate  $K$  vertices to minimize the expected extinction time (i.e., the average time until all individuals have returned to state **S**)

**Challenge**: Infection pathways (edges) are **unknown**

### Our Approach to Solving the VUG Problem

1. **Learn** the underlying graph from observations:  $\hat{\mathcal{G}}$
2. Compute the  $K$  vertices to **vaccinate** using learned graph  $\hat{\mathcal{G}}$



## Learning the Graph: SISLearn

**Observations**:

- Infection states (**S** or **I**) of vertices over rounds  $t \in \{1, 2, \dots, T\}$
- Edges are not observed

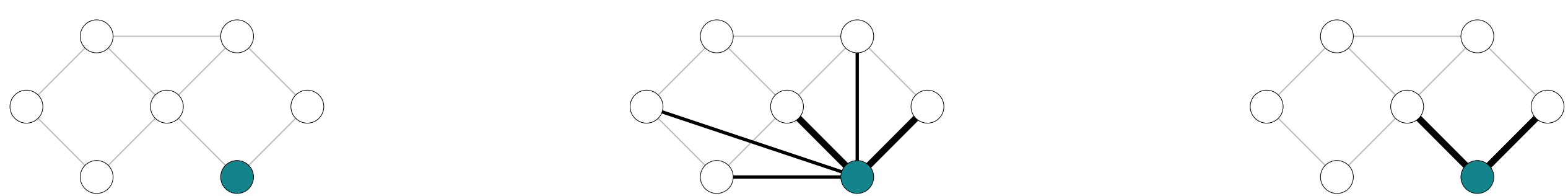
**Idea**: Learn **neighbors** of vertex  $v$  using a correlation indicator

**Correlation Indicator**: Does a vertex  $u$  influence the state of  $v$ ?

$$\mathbb{P}(v \text{ gets infected} \mid u \text{ is infected})$$

**Learning algorithm**: Vertex-wise **inclusion/exclusion mechanism**

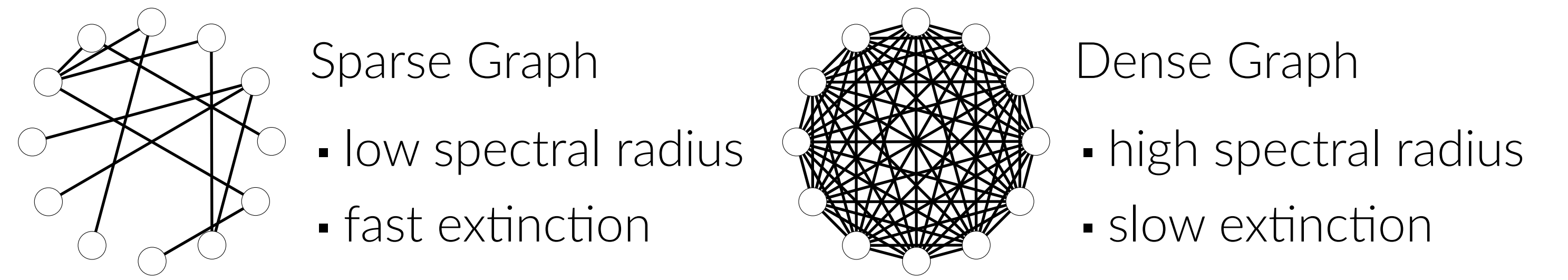
1. Build super-neighborhood from vertices with high influence on  $v$
2. Condition on the super-neighborhood and remove all vertices that do not influence  $v$



## Vaccination Strategies

**Observation**: Minimizing extinction time  $\approx$  minimizing **spectral radius** of the graph  $\rho(\mathcal{G}) := \max\{\lambda \mid \lambda \in \text{eigval}(\mathcal{G})\}$  [1]

**Spectral Radius**: one-number measure of a network's connectivity (higher  $\Rightarrow$  outbreak spreads easier and faster)



### Spectral Radius Minimization Problem (SRM)

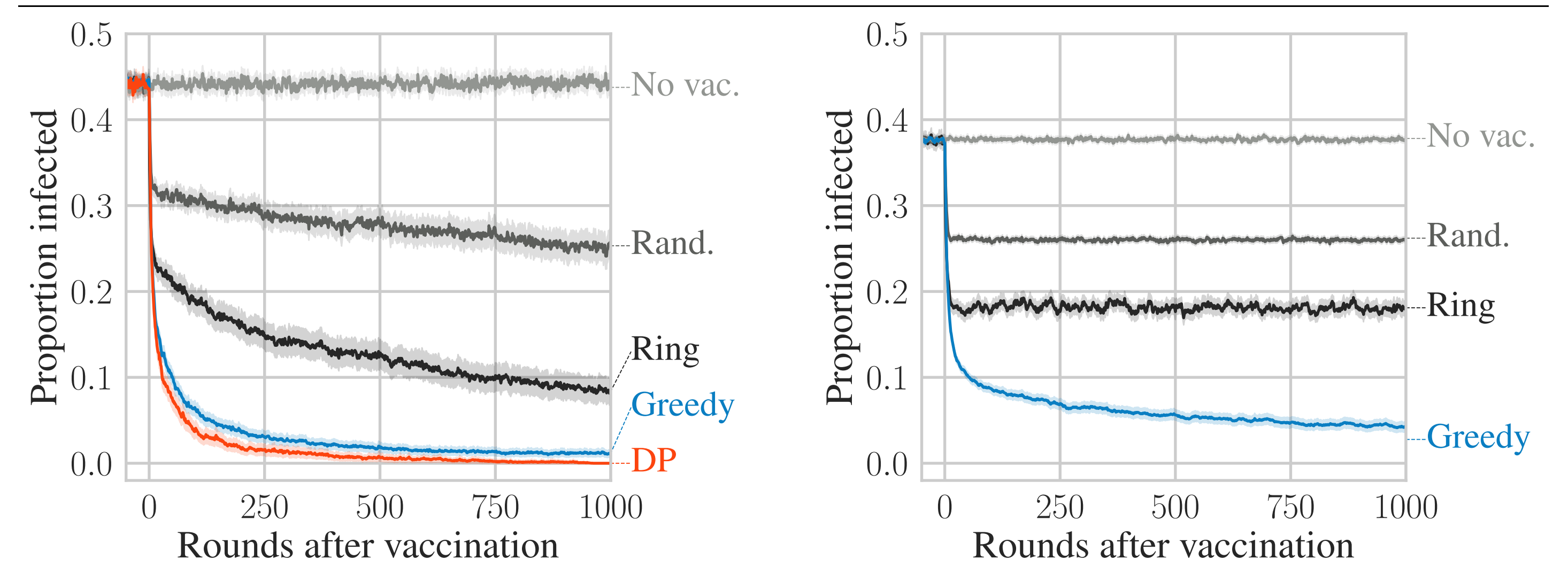
**Idea**: We pick  $K$  people whose vaccination optimally reduces the network's connectivity:  $R^* = \text{argmin}_{R \subseteq V, |R| \leq K} \rho(\mathcal{G}[V \setminus R])$

**Challenge**: SRM is NP-hard on general graphs (naïvely takes exponential time to compute)

**Our Two Approaches**:

- **Dynamic Programming (DP)**: Exact but slower approach (best for networks with  $\leq 60$  vertices)
- **Greedy**: Approximate but fast approach (works on networks with as many as 10'000s vertices)

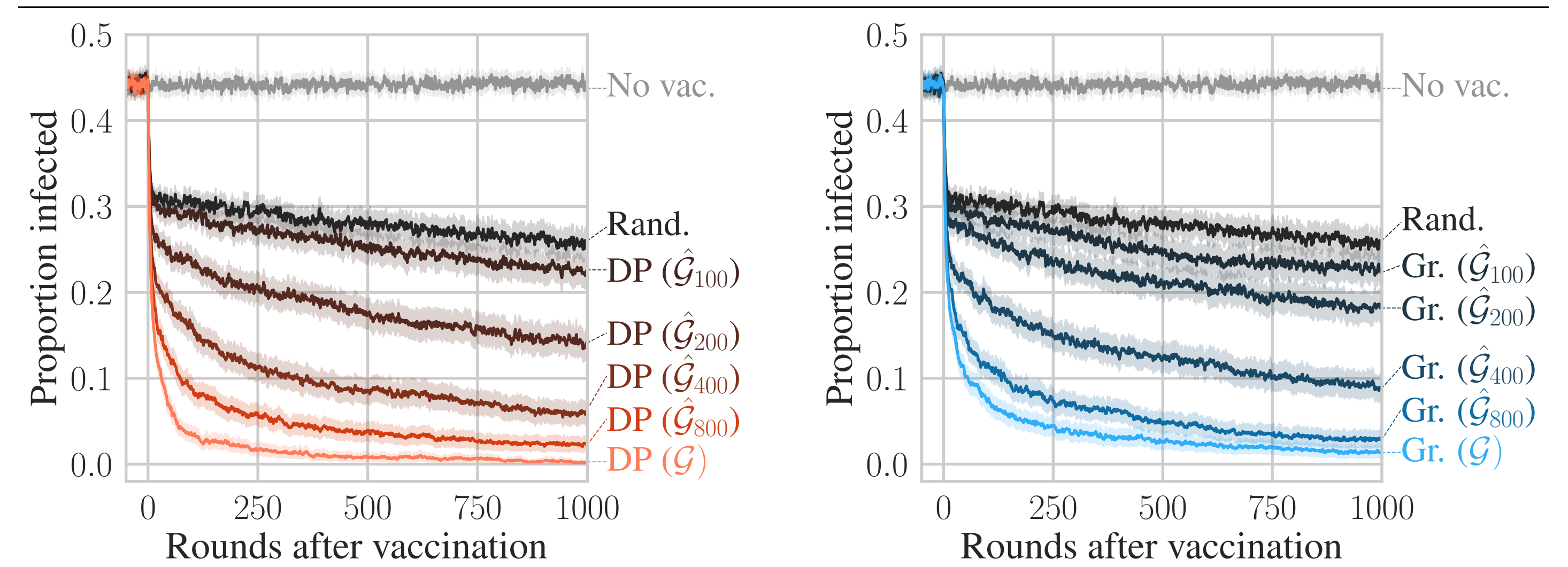
## Performance on Flu Outbreak Networks



DP and Greedy vs. baselines on augmented networks from the 2009 H1N1/H3N2 outbreak in Beijing (40 vertices and 80 edges) [2], learned using SISLearn ( $\downarrow$  is better)

Greedy vs. baselines on augmented networks from the 2009 H1N1 outbreak in Pennsylvania (286 vertices and 818 edges) [2], learned using SISLearn ( $\downarrow$  is better)

## Performance with Limited Data



DP on the learned graph ( $\hat{\mathcal{G}}_{T'}$ ) from SISLearn using different numbers of rounds ( $T'$ ) for learning augmented 2009 Beijing H1N1/H3N2 networks ( $\downarrow$  is better)

Greedy on the learned graph ( $\hat{\mathcal{G}}_{T'}$ ) from SISLearn using different numbers of rounds ( $T'$ ) for learning augmented 2009 Beijing H1N1/H3N2 networks ( $\downarrow$  is better)

## References

- [1] P. Van Mieghem, D. Stevanović, F. Kuipers, C. Li, R. van de Bovenkamp, D. Liu, and H. Wang, "Decreasing the spectral radius of a graph by link removals," *Phys. Rev. E*, 2011.
- [2] J. C. Taube, P. B. Miller, and J. M. Drake, "An open-access database of infectious disease transmission trees to explore superspreader epidemiology," *PLOS Biology*, 2022.

