



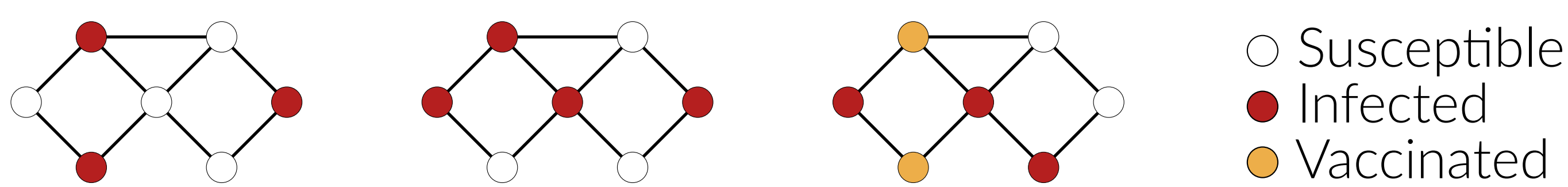
Motivation

Mathematically rigorous approach to contact network recovery and vaccination in a reinfecting epidemic model, posing two **challenges**:

- **Unknown Network**: The underlying contact graph showing *who can infect whom* is unknown
- **Limited Resources**: We can only vaccinate a small fraction of the population, so vaccinations must be precise and impactful

The Propagation Model: Graphical SIS

- Individuals represented as vertices in a graph $\mathcal{G}$ with states Susceptible or Infected
- Edges represent all *potential* infection pathways
- Recovery ($I \rightarrow S$) with probability p_{rec}
- Infection ($S \rightarrow I$) with probability $\propto p_{\text{inf}}$ if neighbors are infected
- Vaccinated vertices: lower probability of infection



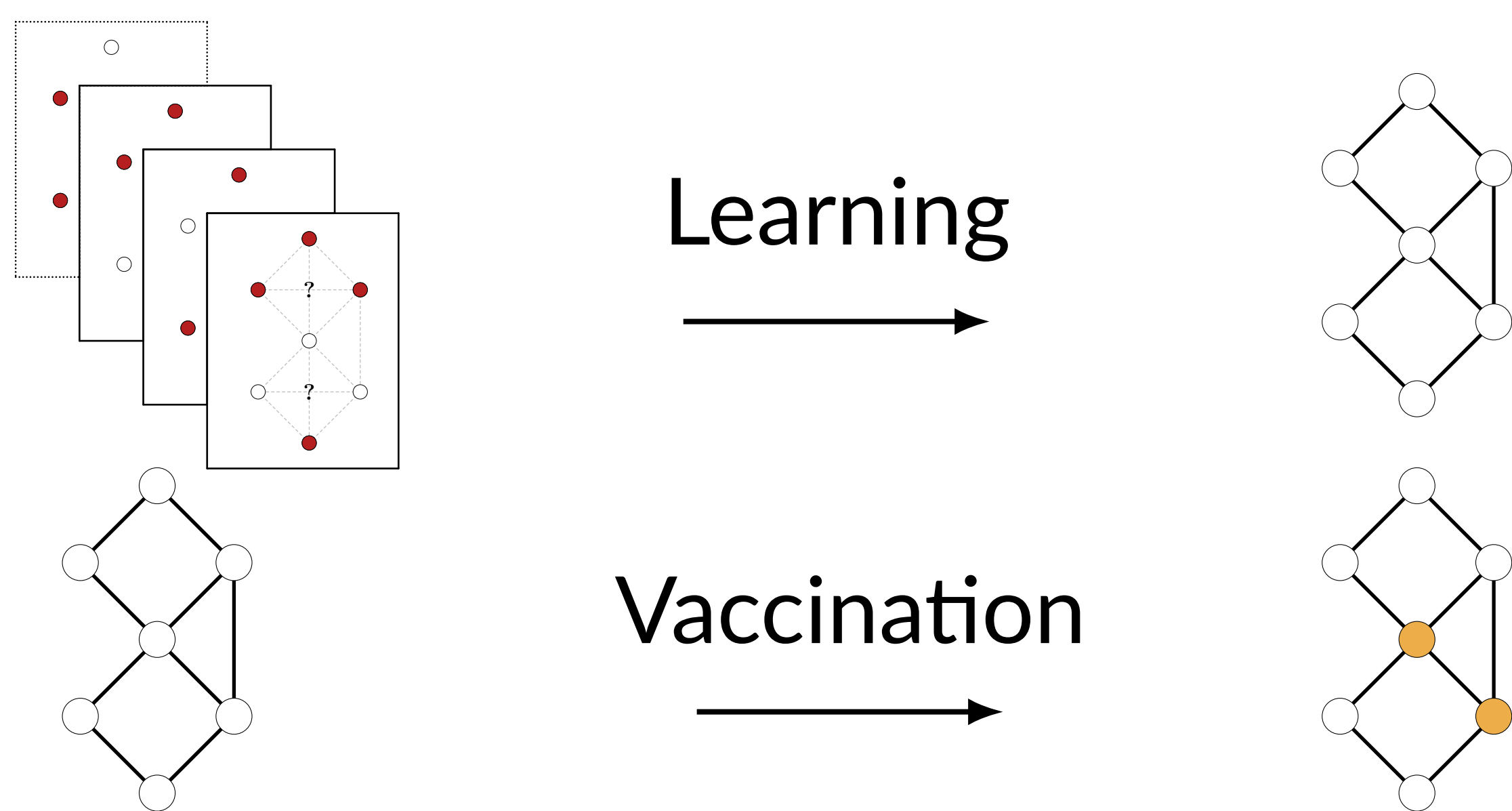
Vaccinating an Unknown Graph (VUG) Problem

Goal: Vaccinate K vertices to minimize the expected extinction time (i.e., the average time until all individuals have returned to state S)

Challenge: Infection pathways (edges) are **unknown**

Our Approach to Solving the VUG Problem

1. **Learn** the underlying graph from observations: $\hat{\mathcal{G}}$
2. Compute the K vertices to **vaccinate** using learned graph $\hat{\mathcal{G}}$



Learning the Graph: SISLearn

Observations:

- Infection states (S or I) of vertices over rounds $t \in \{1, 2, \dots, T\}$
- Edges are not observed

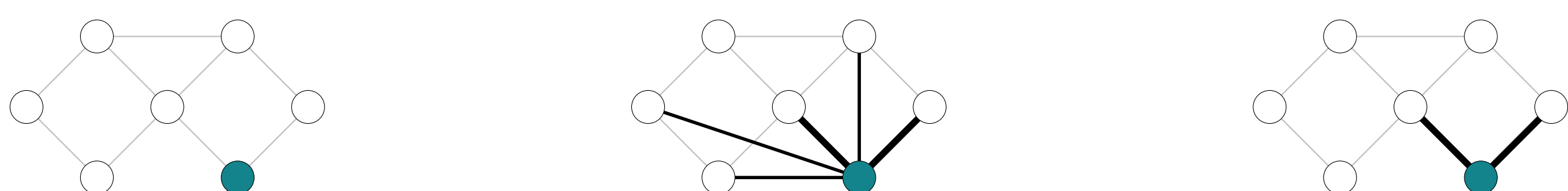
Idea: Learn **neighbors** of vertex v using a correlation indicator

Correlation Indicator: Does a vertex u **influence** the state of v ?

$$\mathbb{P}(v \text{ gets infected} \mid u \text{ is infected})$$

Learning algorithm: Vertex-wise **inclusion/exclusion mechanism**

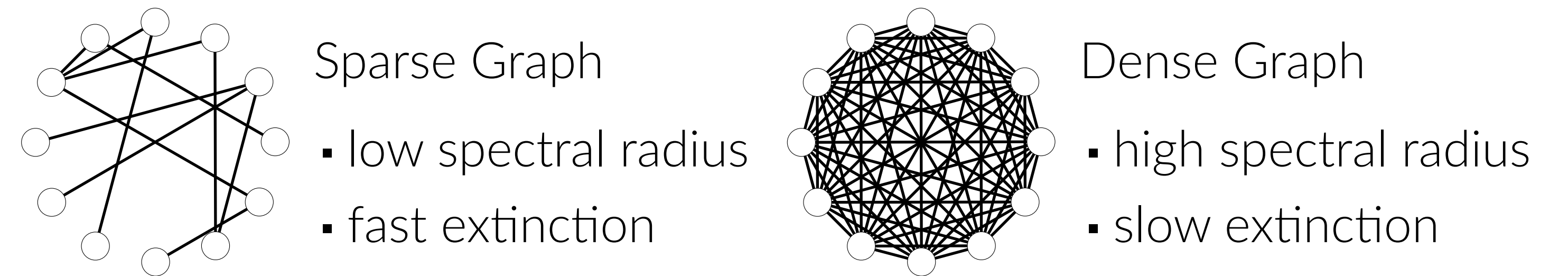
1. Build super-neighborhood from vertices with high influence on v
2. Condition on the super-neighborhood and remove all vertices that do not influence v



Vaccination Strategies

Observation: Minimizing extinction time $\approx$ minimizing **spectral radius** of the graph $\rho(\mathcal{G}) := \max\{\lambda \mid \lambda \in \text{eigval}(\mathcal{G})\}$ [1]

Spectral Radius: one-number measure of a network's connectivity (higher $\Rightarrow$ outbreak spreads easier and faster)



Spectral Radius Minimization Problem (SRM)

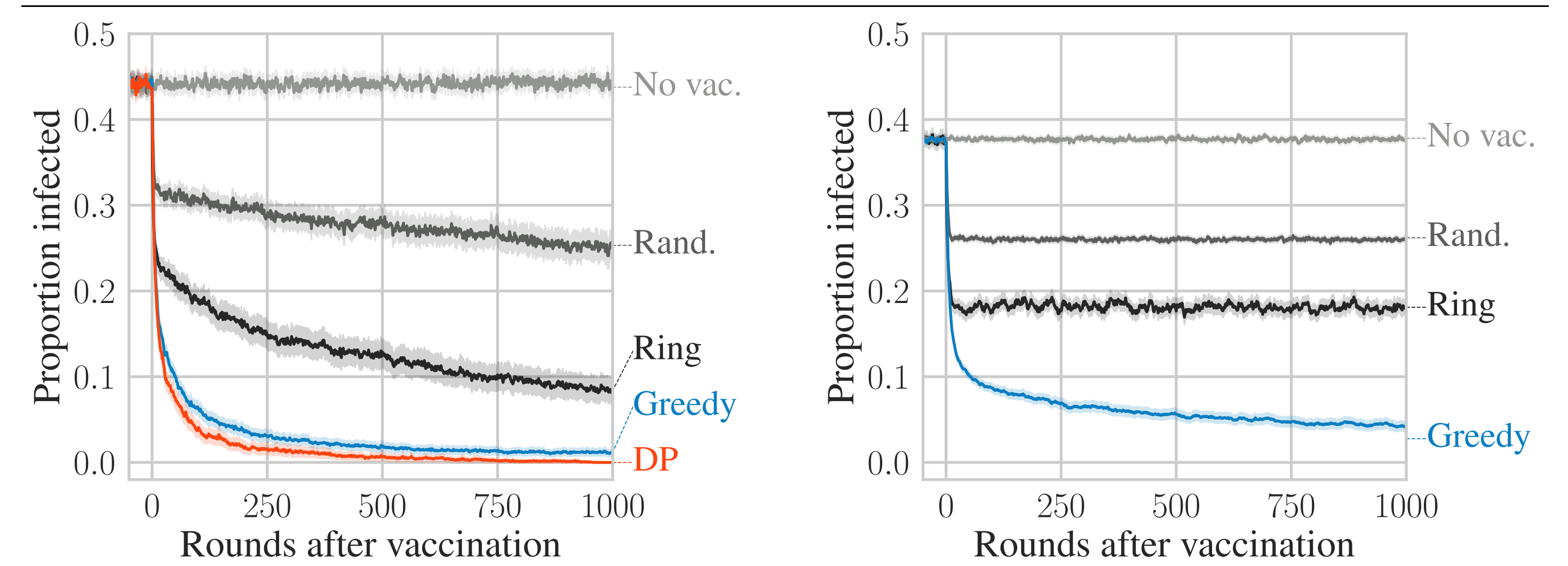
Idea: We pick K people whose vaccination optimally reduces the network's connectivity: $R^* = \text{argmin}_{R \subseteq V, |R| \leq K} \rho(\mathcal{G}[V \setminus R])$

Challenge: SRM is NP-hard on general graphs (naïvely takes exponential time to compute)

Our Two Approaches:

- **Dynamic Programming (DP)**: Exact but slower approach (best for networks with ≤ 60 vertices)
- **Greedy**: Approximate but fast approach (works on networks with as many as 10'000s vertices)

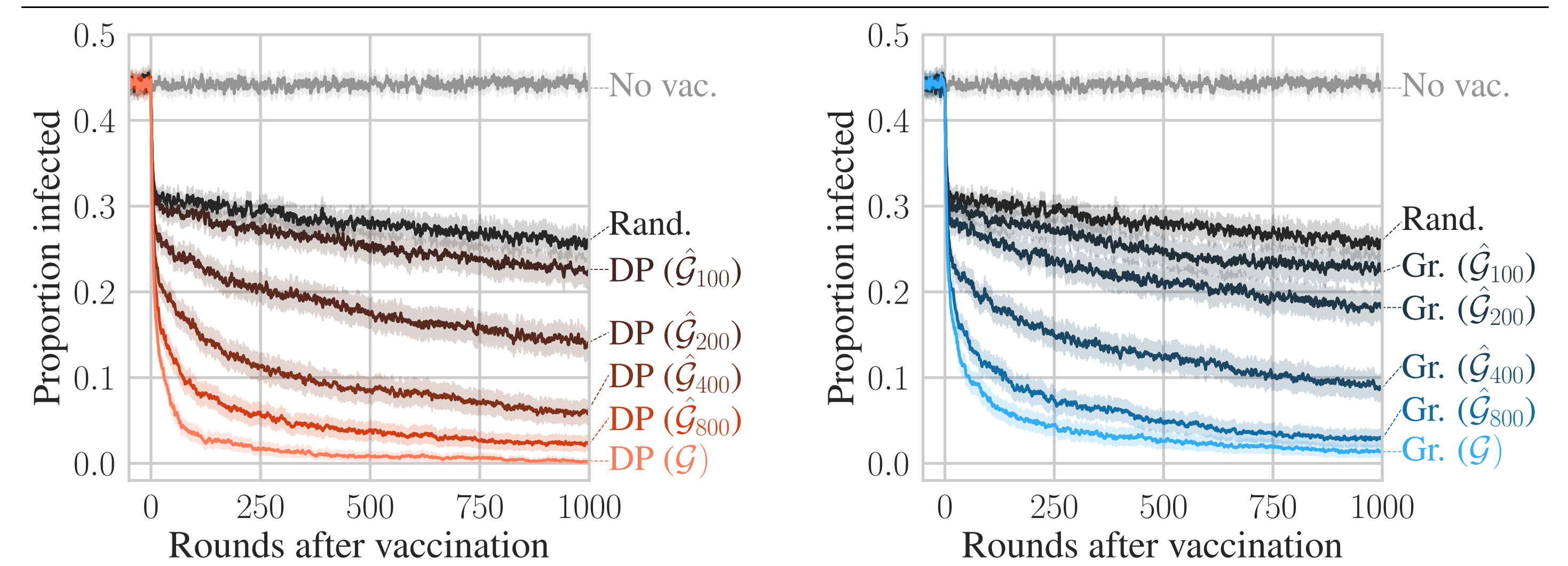
Performance on Flu Outbreak Networks



DP and Greedy vs. baselines on augmented networks from the 2009 H1N1/H3N2 outbreak in Beijing (40 vertices and 80 edges) [2], learned using SISLearn ($\downarrow$ is better)

Greedy vs. baselines on augmented networks from the 2009 H1N1 outbreak in Pennsylvania (286 vertices and 818 edges) [2], learned using SISLearn ($\downarrow$ is better)

Performance with Limited Data



DP on the learned graph ($\hat{\mathcal{G}}_{T'}$) from SISLearn using different numbers of rounds (T') for learning augmented 2009 Beijing H1N1/H3N2 networks ($\downarrow$ is better)

Greedy on the learned graph ($\hat{\mathcal{G}}_{T'}$) from SISLearn using different numbers of rounds (T') for learning augmented 2009 Beijing H1N1/H3N2 networks ($\downarrow$ is better)

References

- [1] P. Van Mieghem, D. Stevanović, F. Kuipers, C. Li, R. van de Bovenkamp, D. Liu, and H. Wang, "Decreasing the spectral radius of a graph by link removals," *Phys. Rev. E*, 2011.
- [2] J. C. Taube, P. B. Miller, and J. M. Drake, "An open-access database of infectious disease transmission trees to explore superspreader epidemiology," *PLOS Biology*, 2022.

