Learn to Vaccinate: Combining Structure Learning and Effective Vaccination for Epidemic and Outbreak Control

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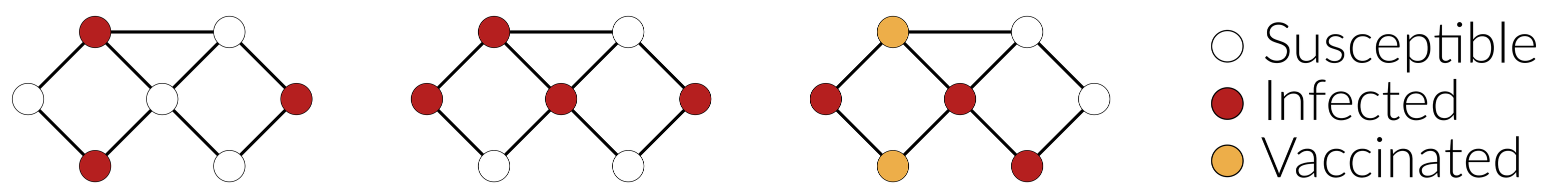
Motivation

Controlling outbreaks (e.g., epidemics, misinformation, etc.) requires targeted interventions, but there are two main **challenges**:

- Unknown Network: The underlying contact graph showing *who can infect whom* is rarely known
- Limited Resources: We can only vaccinate a small fraction of the population, so interventions must be precise and impactful

The Propagation Model: SIS

- Underlying graph $\mathcal{G}$ with vertex states **S**usceptible or **I**nfected
- Edges represent all *potential* infection pathways
- Recovery (I $\to$ S) with prob. p_{rec}
- Infection (S $\to$ I) with prob. $\propto p_{inf}$ if neighbors are infected
- Vaccinated vertices: lower prob. of infection



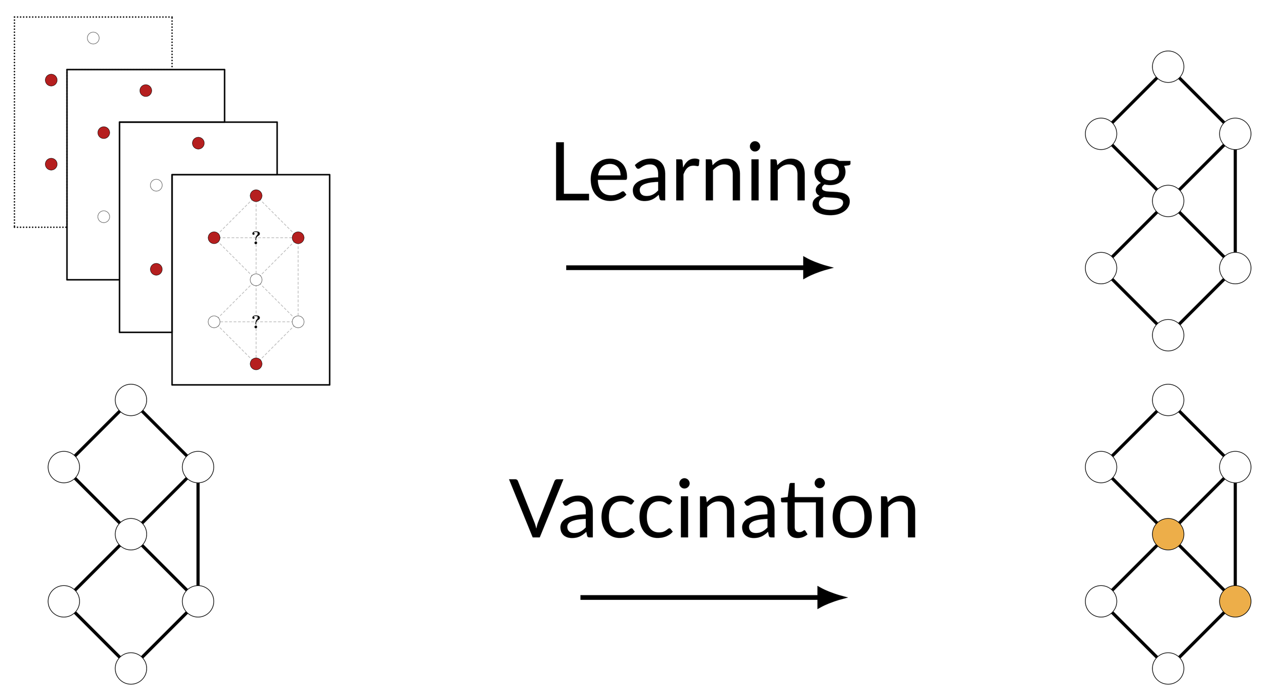
Vaccinating an Unknown Graph Problem

Goal: **Vaccinate** K vertices to min. the expected extinction time

Challenge: Infection pathways (edges) are **unknown**

Our Approach to Solving the VUG Problem

1. **Learn** the underlying graph from observations: $\hat{\mathcal{G}}$
2. Compute the K vertices to **vaccinate** using learned graph $\hat{\mathcal{G}}$



Learning the Graph: `SISLearn`

Observations:

- Infection states of vertices over rounds $t \in \{1, 2, \ldots, T\}$
- Edges are not observed

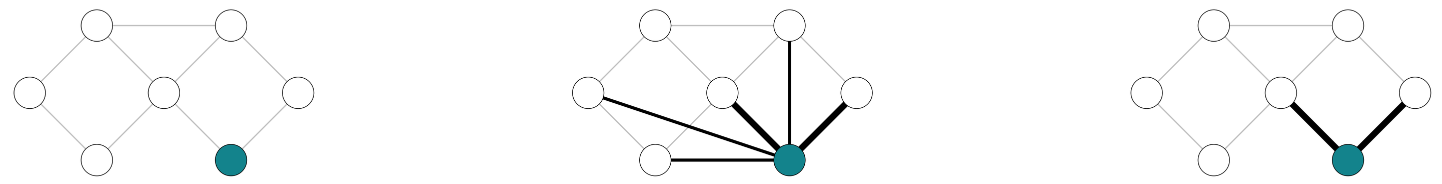
Idea: Learn **neighbors** of vertex v using a correlation indicator [1]

Correlation Indicator: Does a vertex u **influence** the state of v?

$$\mathbb{P}(v \text{ gets infected } \mid u \text{ is infected})$$

Learning algorithm: Vertex-wise **inclusion/exclusion mechanism**

1. Build super-neighborhood from vertices with high influence on v
2. Condition on the super-neighborhood and remove all vertices that do not influence v



Vaccination Strategies

Observation: Minimizing extinction time $\approxeq$ minimizing **spectral radius** of the graph $\rho(\mathcal{G}) := \max\{\lambda \mid \lambda \in \text{eigval}(\mathcal{G})\}$ [2]

Spectral Radius Minimization Problem (SRM)

Idea: Find vaccination set R^* using the SRM surrogate problem:

$$R^* = \operatorname*{argmin}_{R \subseteq V, |R| \leq K} \rho(\mathcal{G}[V \setminus R])$$

Challenge: SRM is NP-hard on general graphs

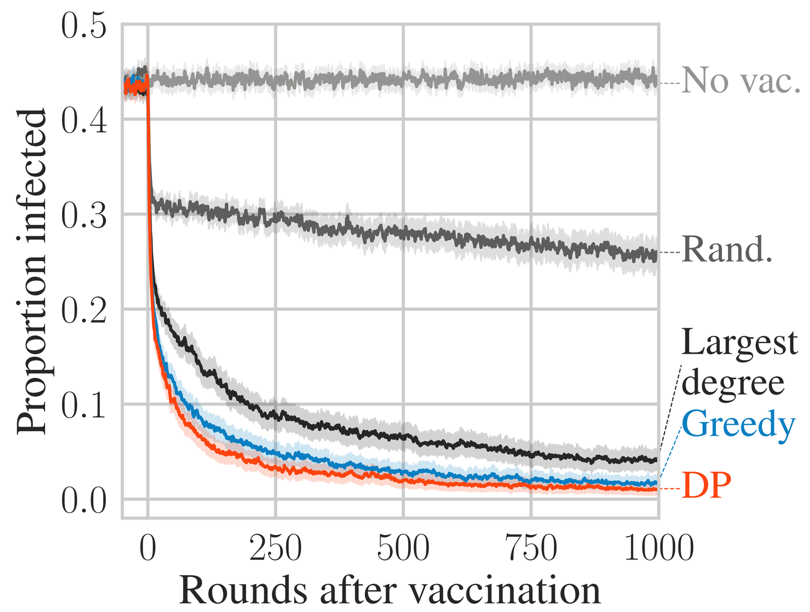
Exact approach: Dynamic Programming (DP) algorithm on the tree decomposition to compute the optimal vaccination set R^*

Heuristic approach: Greedy algorithm that iteratively vaccinates the vertex that reduces the spectral radius the most

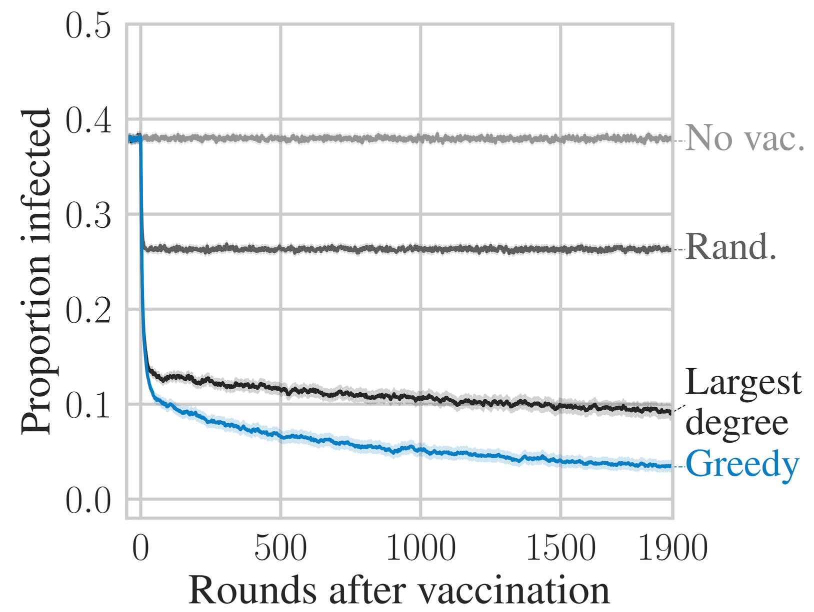
Approach Name	Optimal on SRM	Time Complexity
DP	✓	$\mathcal{O}(n^3 K^3 2^\omega)$
Greedy	✗	$\mathcal{O}(n^3 K)$

where ω is the treewidth of the graph $\mathcal{G}$

Performance on Flu Outbreak Networks

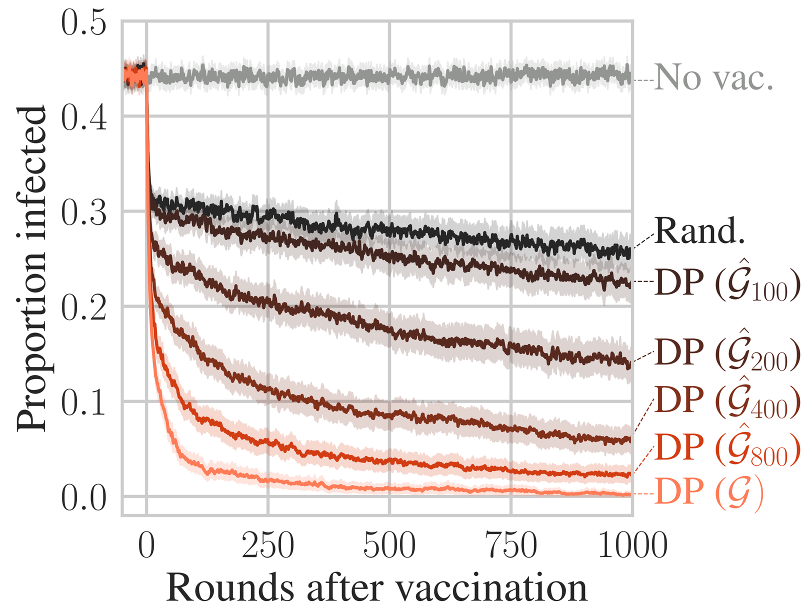


DP and Greedy vs. baselines on augmented networks from the 2009 H1N1/H3N2 outbreak in Beijing (40 vertices and 80 edges) [3], learned using `SISLearn` (↓ is better)

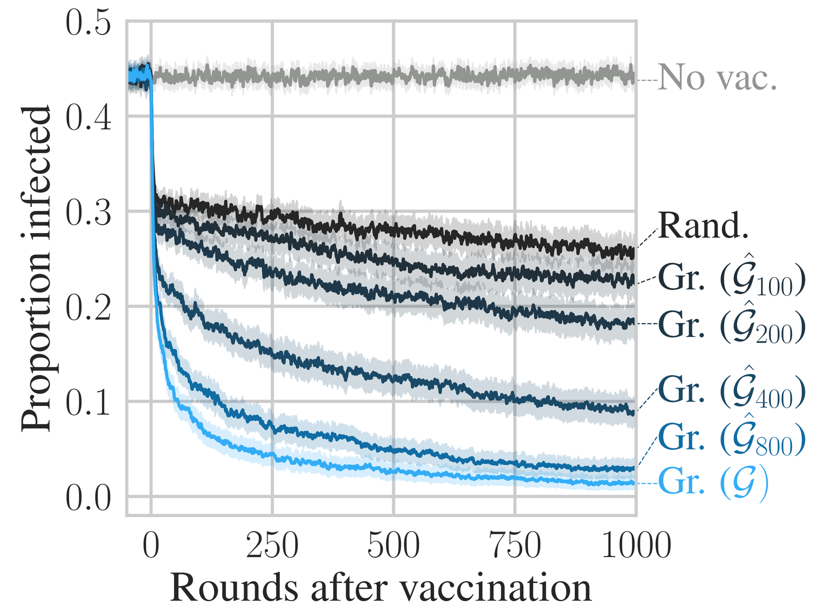


Greedy vs. baselines on augmented networks from the 2009 H1N1 outbreak in Pennsylvania (286 vertices and 818 edges) [3], learned using `SISLearn` (↓ is better)

Performance with Limited Observations



DP on the learned graph ($\hat{\mathcal{G}}_{T'}$) from `SISLearn` using different numbers of rounds (T') for learning augmented 2009 Beijing H1N1/H3N2 networks (↓ is better)



Greedy on the learned graph ($\hat{\mathcal{G}}_{T'}$) from `SISLearn` using different numbers of rounds (T') for learning augmented 2009 Beijing H1N1/H3N2 networks (↓ is better)

More Information & References

[1] G. Bresler, "Efficiently learning ising models on arbitrary graphs," in *STOC '15*, 2015.

[2] P. Van Mieghem, D. Stevanović, F. Kuipers, C. Li, R. van de Bovenkamp, D. Liu, and H. Wang, "Decreasing the spectral radius of a graph by link removals," *Phys. Rev. E*, 2011.

[3] J. C. Taube, P. B. Miller, and J. M. Drake, "An open-access database of infectious disease transmission trees to explore superspreader epidemiology," *PLOS Biology*, 2022.

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